

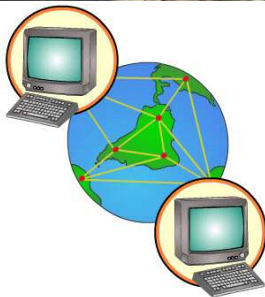
FAST: Theory and practice of acceleration

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24 avril 2006

Verification of reactive systems



Reactive systems

- Software and/or hardware
- Autonomous
- **Critical**

The TTP protocol



Processors embedded in cars

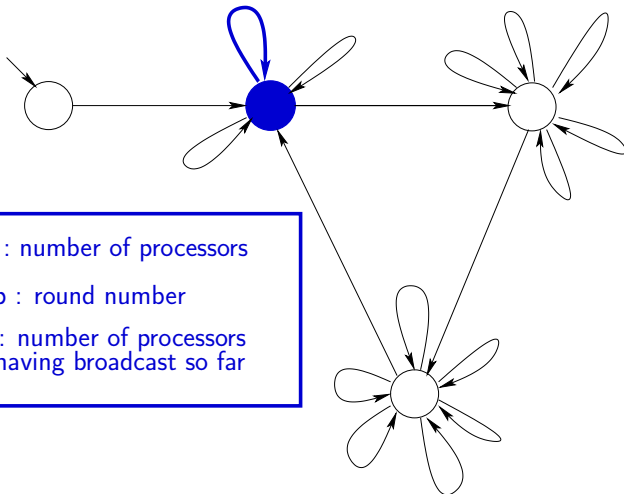
The TTP protocol

- fault tolerance
- ensure no fault will propagate

TTP is supported by Audi, PSA, Renault, ...

A model of the TTP [Bouajjani-Merceron 2002]

```
If  $d < N$  Do  
   $d := d + 1; C_p := C_p + 1$   
End Do
```

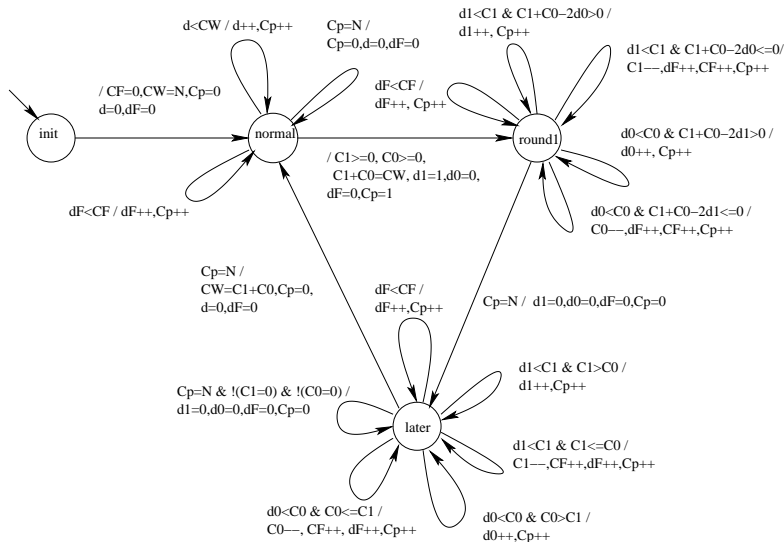


N : number of processors

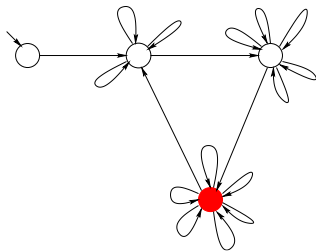
C_p : round number

d : number of processors
having broadcast so far

A model of the TTP [Bouajjani-Merceron 2002]



A model of the TTP [Bouajjani-Merceron 2002]



Question

In the **red location**, does
 $C_p = N \Rightarrow (C_0 = 0 \vee C_1 = 0)$?

Objective

Automatic verification for **any value** of N

Our problem: counter system verification

Counter systems

- we study mathematical models of concrete systems
- automata extended with **unbounded integer variables**

Properties to check

Reachability properties = properties of reachable configurations.

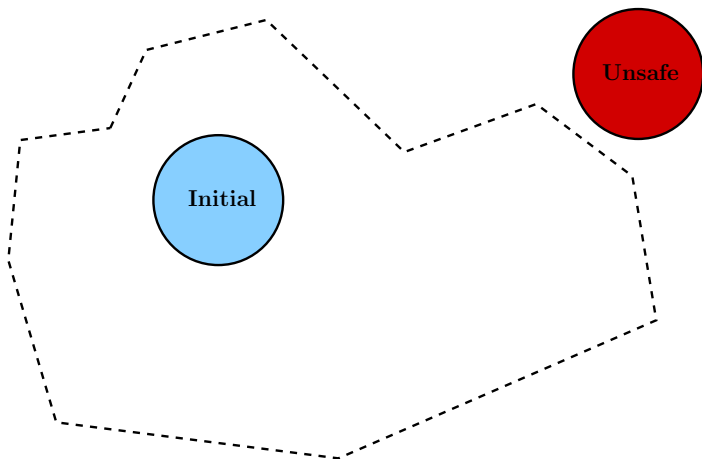
- useful: mutual exclusion, deadlock freedom, ...
- easy to check from the reachability set.

Our problem: counter system verification

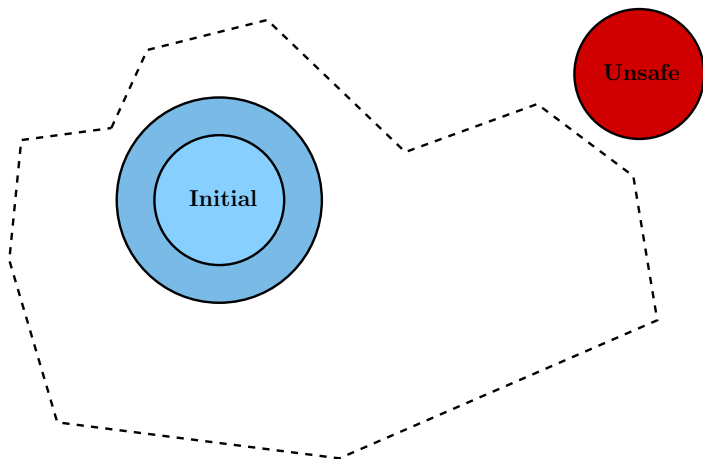
Problems

- Undecidable for two counters with $(+1, -1, \stackrel{?}{=} 0)$
- One of the issues: **infinite** reachability set

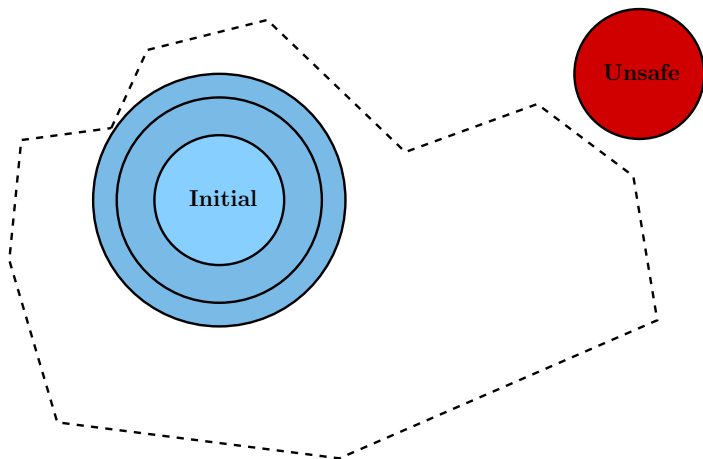
Back to the finite case



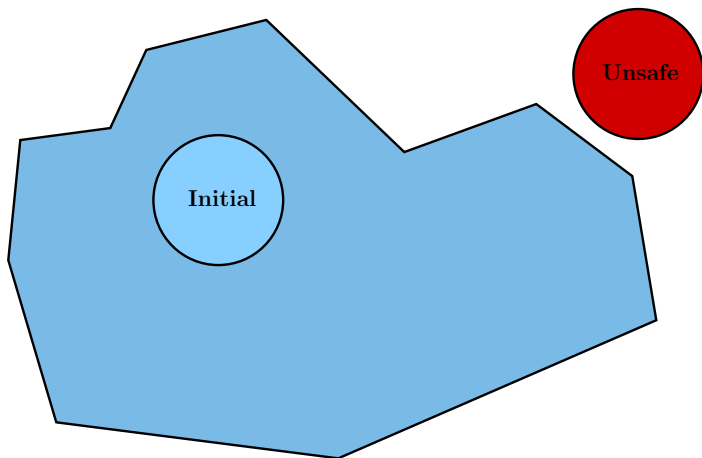
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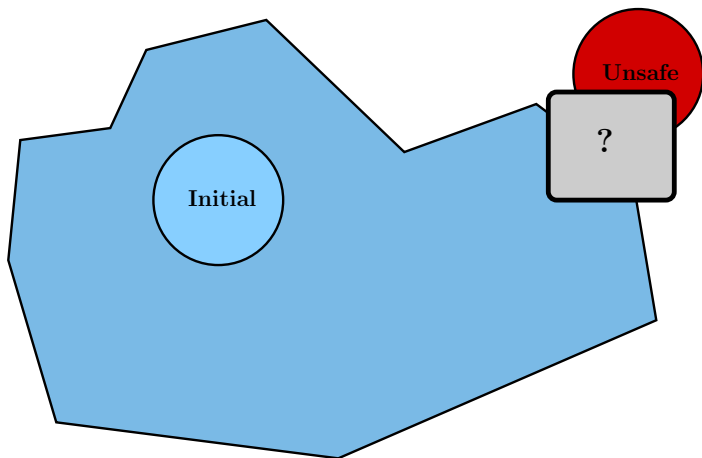
Back to the finite case



Back to the finite case



Back to the finite case



Enumerative methods do not work any more

Algorithms for decidable subclasses

- Petri nets,
- timed automata, ...

or **Semi-algorithms to compute the reachability set**

- more expressive/realistic systems
- no guarantee of termination, we hope **practical termination**
- Extend iterative computation for infinite sets
- (**symbolic model-checking**)

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Issue 1: infinite set of reachable configurations.

Idea = manipulate infinite sets of configurations

- sets are represented **symbolically**.
- need basic symbolic operations POST , \sqcup , \sqsubseteq .

Example: intervals of integers

- Formula $\phi_x : \{x > 5\}$ means that x ranges over all integers greater than 5
- After transition $\xrightarrow{y:=x+1}$, the possible values of y are exactly represented by $\phi_y = \{y > 6\}$

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First (and basic) symbolic procedure

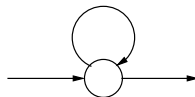
Iterative computation of $\text{post}_S^*(X_0)$

- 1 $X \leftarrow X_0$
- 2 If $\text{POST}(X) \subseteq X$ Goto 5
- 3 $X \leftarrow \text{POST}(X) \sqcup X$
- 4 Goto 2
- 5 Return X

Issue 2: termination is scarce

because of circuits in the control graph ...

If $x \geq 0$ Do $x \leftarrow x + 2$



If $X_0 = \{0\}$ then

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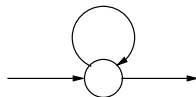
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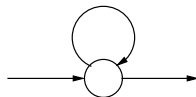
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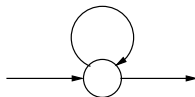
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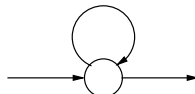
If $X_0 = \{0\}$ then $X = \{0, 2, \dots, 2k\}$

Principle of circuit acceleration

Circuit acceleration

Enhance the convergence of the iterative symbolic procedure by computing in one step **the iteration** of a **sequence of transitions** (circuit).

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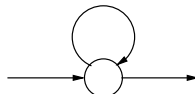


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If $x \geq 0$ Do $x \leftarrow x + 2$



If $X_0 = \{0\}$ then $\text{post}^*(X_0) = 2.\mathbb{N}$, in one step.

About acceleration of counter systems

State-of-the-art in 2002

(*Karp-Miller 1969*)

(*Fribourg 1990*)

[Boigelot-Wolper 1994],

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Remarks

- 1 very different techniques, no unifying view (comparison?)
- 2 still a gap between acceleration algorithm and fixpoint computation (how to select circuits?)

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- A symbolic representation: automata
 - DFA [Boudet-Comon 1996],
 - NDD [Wolper-Boigelot 2000]).
- Various circuit acceleration algorithms
 - $f(x) = M.x + v$, with *finite monoid* and convex guard [Boigelot 1998]
 - $f(x) = M.x + v$ with *finite monoid* and Presburger guard [Finkel-Leroux 2002]
 - functions “à la” timed automata [Annichini-Asarin-Bouajjani 2000]
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- Tools
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Issue : Various acceleration techniques

Results : Unifying framework encompassing most of acceleration theorems [ATVA'05]

Issue : How to select circuits?

Results : Maximal heuristic, efficient in practice [CAV'03,ATVA'05]

Issue : Improve practical efficiency of acceleration

Results : “Convex acceleration” algorithm [TACAS'04]

Issue : Experimentations

Results : implementation of FAST [CAV'03],
Verification of various protocols: TTP [TACAS'04] , CES
and others.

These works have been partially supported by ACI PERSÉE.

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- 3 Circuit acceleration
- 4 Circuit selection
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Presburger arithmetics

First order arithmetics without \times -operator

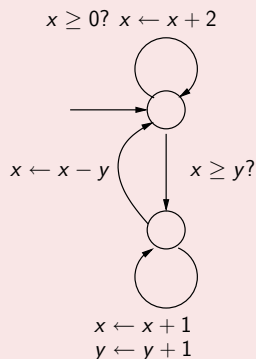
$$\phi ::= t \leq t \mid \neg \phi \mid \phi \vee \phi \mid \exists k. \phi \mid \text{true}$$

$$t ::= 0 \mid 1 \mid y \mid t - t \mid t + t.$$

Presburger set = set of solutions of a Presburger formula.

Counter systems

- finite set of ***m* variables** x, y, z, \dots over \mathbb{N}
- finite set of P-affine functions $f = (M, v, G)$
 - $G \subseteq \mathbb{N}^m$ Presburger guard
 - M square matrix
 - v vector
- $\vec{var}' = f(\vec{var})$ iff
 - $\vec{var} \in G$
 - and $\vec{var}' = M \cdot \vec{var} + v$



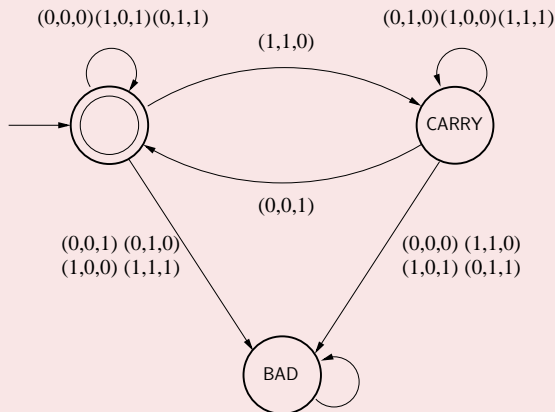
Automata to recognize sets of integer vectors

- a non-negative integer in basis 2 is a word over $\{0, 1\}$
- automata recognize sets of words
- extensions
 - any integer: 2-complement encoding
 - vectors: tuples of letters, or variables entanglement

Presburger sets (and a little bit more) are recognized by automata.

Common operations on sets \longrightarrow standard operations on automata

Symbolic representations: Automata



- DFA [Boudet-Comon 1996],
- NDD [Wolper-Boigelot 2000]).

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Monoid of a function $f = (M, v, G): \{1, M, M^2, \dots, M^n, \dots\}$

Theorem [Finkel-Leroux 2002]

Let $f = (M, v, G)$ a P-affine function with **finite monoid**. Then f^* is effectively defined by a Presburger formula

$$f^* = \{(x, x') \mid x \in G \wedge \exists k \geq 0. x' = \bar{f}^k(x) \wedge \forall i. 0 \leq i < k, \bar{f}^i(x) \in G\}$$

Building the formula is 3-EXP in $|\mathcal{A}(G)|$, $|v|_{\max}$, $|M|_{\max}$ et m .

Idea of the algorithm

- $f = (M, v, G)$ with finite monoid $\langle M \rangle$.
- $\bar{f} : \mathbb{Z}^m \rightarrow \mathbb{Z}^m, \forall x \in \mathbb{Z}^m, \bar{f}(x) = M.x + v$

- $\langle M \rangle$ finite, then $\exists(a, b) \in \mathbb{N} \times \mathbb{N}$ such that $M^{a+b} = M^a$
- We deduce that $\forall n \in \mathbb{N}, \forall x \in \mathbb{Z}^m, \bar{f}^{a+n.b} = \bar{f}^a(x) + n.M^a.\bar{f}^b(0)$
- It comes that $\bar{F} = \{(i, x, x') \in \mathbb{N} \times \mathbb{Z}^m \times \mathbb{Z}^m, x' = \bar{f}^i(x)\} \iff \bigvee_{r=0}^{a-1} \{(i, x, x') | x' = \bar{f}^r(x) \wedge i = r\} \bigvee_{r=0}^{b-1} \{(i, x, x') | \exists n \geq 0 (x' = \bar{f}^{a+r}(x) + n.M^a.\bar{f}^b(0)) \wedge (i = a + r + n.b)\}$

$$f^* = \{(x, x'), \exists i \geq 0, x' = f^i(x)\} \iff \{(x, x'), \exists i \geq 0 [(i, x, x') \in \bar{F} \wedge (\forall k (0 \leq k < i), \exists x'' \in G, (k, x, x'') \in \bar{F})]\}$$

Convex translations [TACAS'04]

$f = (I_m, v, G)$ where I_m is the **identity matrix** and G **convex**

- No need to test if the predecessors are in the guard.
- The construction can be simplified.

Theorem [TACAS'04]

Convex acceleration is quadratic in $|\mathcal{A}(G)|$.

Faster acceleration ... complexity

parameter	magnitude	standard algorithm	convex algorithm
$ A(G) $	100.000	3-EXP	quadratic
m	5-50	3-EXP	EXP
$ V _{max}$	≤ 10	3-EXP	poly. in m
$ M _{max}$	≤ 10	3-EXP	$= 1$

Faster acceleration ... practice

$\mathcal{A}(f^*)$ = automaton representing f^* (transductor)

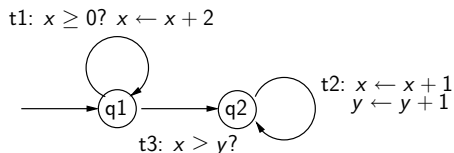
$ \mathcal{A}(f^*) $	Time (seconds) Standard/Convex	Memory (MB) Standard/Convex
16,766	10/7	31/13
26,409	5/2	17/18
41,950	18/10	52/30
190,986 (TTP2)	50/9	400/140
380,332 (TTP2)	↑↑↑/34	↑↑↑/534
?	↑↑↑/ >900	↑↑↑/ >500

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What is computable with circuit acceleration?

First answer

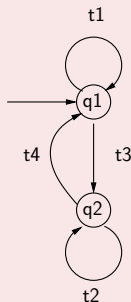
Flat system = at most 1 elementary circuit on each control node



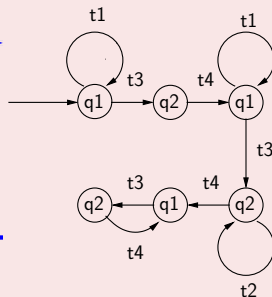
Circuit acceleration + flat S = computation of $\text{post}_S^*(X_0)$

Flattenings of non-flat systems

Système S



Système S'



S' is a **flattening** of S iff
 S' is flat and S simulates S'

(S, X_0) is flattable iff $\exists S'$ such that

- S' is a flattening of S
- S and S' are equivalent for reachability.

Theorem 1 [ATVA'05]

$\text{post}^*(X_0)$ is computable by circuit acceleration iff (S, X_0) is flattable.

A restricted linear regular expression (rlre) over T

$$\rho = w_1^* w_2^* \dots w_n^*, \text{ where } w_i \in T^*.$$

$\text{post}(\rho, X) =$ configurations reachable following transitions in ρ

Theorem 2 [ATVA'05]

$\text{post}^*(X_0)$ is computable by circuit acceleration iff \exists a rlre ρ over T such that $\text{post}^*(X_0) = \text{post}(\rho, X_0)$.

Remark: $\text{post}(\rho, X)$ is computable with circuit acceleration

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Selection Heuristic (I)

Input: (S, X_0)

- 1 $X \leftarrow X_0; k \leftarrow 0$
- 2 $k \leftarrow k + 1$
- 3 **Lunch**
- 4 If $\text{post}(X) \subseteq X$ Goto 10
- 5 Enumerate the next ρ rlre over T
- 6 $X \leftarrow \text{post}(\rho, X)$
- 7 Goto 4
- 8 **In parallel with**
- 9 **When** *Watchdog* stops **Goto** 2
- 10 Return X

maximal procedure: terminates iff (S, X_0) is flattable

PB1(time) find quickly a good rlre

PB2(space) avoid as much as possible unnecessary expensive steps of computations

Selection Heuristic (I)

Input: (S, X_0)

- 1 $X \leftarrow X_0; k \leftarrow 0$
- 2 $k \leftarrow k + 1$
- 3 **Lunch**
- 4 If $\text{post}(X) \subseteq X$ Goto 10
- 5 *Choose fairly* $w \in T^{\leq k}$
- 6 $X \leftarrow \text{post}(w^*, X)$
- 7 Goto 4
- 8 **In parallel with**
- 9 **When** *Watchdog* stops **Goto** 2
- 10 Return X

The procedure is still maximal

PB1(time) partitioning + *Watchdog*

PB2(space) *Choose*

Selection Heuristic (II)

Results

The selection heuristic design is reduced to designing

- *Choose* (a standard solution is given)
- *Watchdog* (a standard solution is given)

The obtained procedure is then

- *maximal*
- *efficient* : good results on counter systems (cf. FAST)

$|T|^{\leq k}$ may be exponential in k .

Idea = reduce $|T|^{\leq k}$ by removing redundant functions.

Three reductions:

- **union-reduction** [Finkel-Leroux 2002]
 - if $f = (M, v, G_1)$ and $g = (M, v, G_2)$,
 - let $h = (M, v, G_1 \vee G_2)$
 - then $(f + g)^* = h^*$
- **commutation-reduction** [CAV'03]
 - if f and g commute then $f^* g^* = (f \cdot g)^* = (g \cdot f)^*$
- **conjugacy-reduction** [ATVA'05]
 - $(f_2 \cdot f_3 \cdot f_1)^* = I_d + f_2 \cdot f_3 \cdot (f_1 \cdot f_2 \cdot f_3)^* \cdot f_1$

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Three reductions:

- **union-reduction** [Finkel-Leroux 2002]
 - if $f = (M, v, G_1)$ and $g = (M, v, G_2)$,
 - let $h = (M, v, G_1 \vee G_2)$
 - then $(f + g)^* = h^*$
- **commutation-reduction** [CAV'03]
 - if f and g commute then $f^* g^* = (f \cdot g)^* = (g \cdot f)^*$
- **conjugacy-reduction** [ATVA'05]
 - $(f_2 \cdot f_3 \cdot f_1)^* = I_d + f_2 \cdot f_3 \cdot (f_1 \cdot f_2 \cdot f_3)^* \cdot f_1$

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Practical results

system	$ T $	k	$ C^{\leq k} $	U	Cm	Cj	U+Cm
csm	13	1	14	14	14	14	14
	13	2	183	103	57	99	35
consistency	8	1	9	9	9	9	9
	8	2	68	45	44	39	30
	8	3	484	172	299	178	98
swimming pool	6	1	7	7	7	7	7
	6	2	43	21	24	25	16
	6	3	259	56	114	97	28
	6	4	1555	126	614	421	47
	6	5	9331	252	3591	1977	86

U, Cm ,Cj : reductions (union, commutation, conjugacy)

- 1 Introduction
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- 3 Circuit acceleration
- 4 Circuit selection
- 5 **The tool FAST**
- 6 Applications
- 7 Conclusion

The previous results are implemented in FAST

FAST works well in practice

- successfully verify 80% of 40 infinite systems [CAV'03].
- first automatic verification of TTP [TACAS'04]
- first automatic verification of CES

Technological comparison

	ALV	LASH	FAST	TREX
system	relational	affine		restricted
symb. rep	automata			arith. + pdbm (undec. \sqsubseteq)
acceleration	no	circuits		circuits (partial.rec.)
circuit selection		no	yes	yes, $\leq k$

Practical comparison

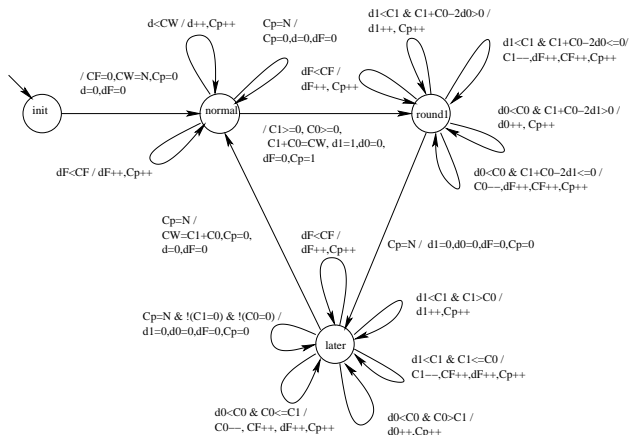
System	ALV	LASH	FAST	k	TREX
RTP (bounded)	T	T	T	1	T
Lamport (bounded)	T	T	T	1	T
Dekker (bounded)	T	T	T	1	T
ticket 2	T	T	T	1	T
kanban	↑	T	T	1	T
multipoll	↑	T	T	1	↑
prod/cons (2)	↑	T	T	1	-
ttp	↑	T	T	1	-
prod/cons (N)	↑	↑	T	2	-
lift control, N	↑	↑	T	2	T
train	↑	↑	T	2	T
csm, N	↑	↑	T	2	↑
consistency	↑	↑	T	3	-
swimming pool	↑	↑	T	4	↑
pncsa	↑	↑	↑	?	↑
incdec	↑	↑	↑	?	↑
bigjava	↑	↑	↑	?	↑

T: success within 20 minutes
↑: no success within 20 minutes

k: circuit length for FAST
-: not an input of TREX

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Verification of TTP by FAST



Verification of TTP by FAST

1 error [TACAS'04]

16 transitions, 9 variables, complex guards

- automatic verification
- Pentium 4 2.4 GHz, 1 Gbyte RAM : 940 sec. and 73 Mbytes.

Other tools:

- ALV does not terminate
- LASH terminates when good circuits are provided
- TTP does not fit TREX input model.

2 errors [TACAS'04]

20 transitions, 18 variables, even more complex guards

- standard acceleration does not work
- convex acceleration + overapproximation.

1 error [TACAS'04]

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The CES protocol - presentation

- Supported by Philips
- multimedia streaming
- ensures reliable communications over lossy channels

Jonathan Billington and Lin Liu [Billington-Liu 2002]

- Colored Petri net modeling of the CES,
- infinite system, counters and **queues of parameterized length**
- (complex) proofs of many properties of the CES (ex: size of the reachability set w.r.t. the buffer lengths)

The CES protocol - verification with FAST

Modeling issues

FAST does not handle queues.

- queues **simulated** by counters,
- **correctness of the simulation** is expressed as a reachability property of the counter system, and it is **checked by FAST** automatically.

Results

Properties proved in [Billington-Liu 2002] are checked easily.

Verification of pointer systems (work in progress)

Manual management of memory resources (language C)

- memory heap = collection of memory cells
- a cell contains: data or address
- addresses $\in \{\text{valid}, \text{invalid}, \text{NULL}\}$
- primitives: `new`, `free`, successor

Common errors

- memory violation
- memory leak

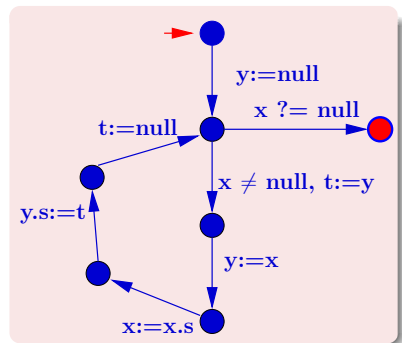
Work supported by EDF (2002-2004),
and by RNTL AVERILES (2005-2008)

Pointer systems

Programs:

- only one successor (lists, no trees)
- no data, only pointers

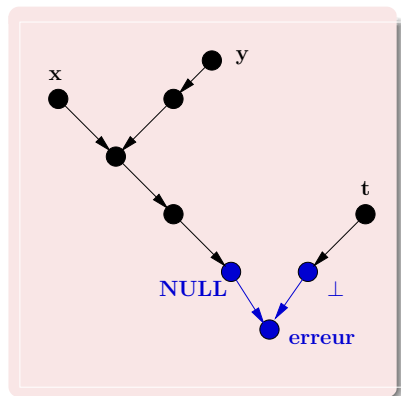
```
List reverse(List x) {  
  List y,t;  
  y =NULL;  
  while (x!=NULL) {  
    t=y;  
    y=x;  
    x=x->n;  
    y->n=t;  
    t=NULL;  
  }  
  return y;  
}
```



Concrete configurations

Memory graphs

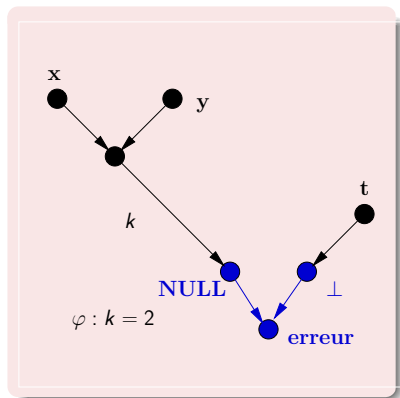
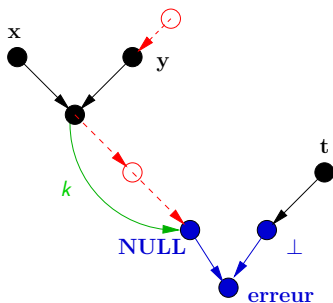
- nodes = memory cells
- edges = “pointed by”
- labels = set of pointer variables pointing the cell
- \perp = invalid addresses



Symbolic representation [AVIS'04]

memory graph (shape) + counters + constraint

- canonical form of shapes
- finite number of shapes



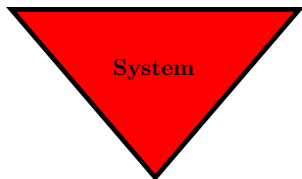
Verification of pointer systems [AVIS'06]

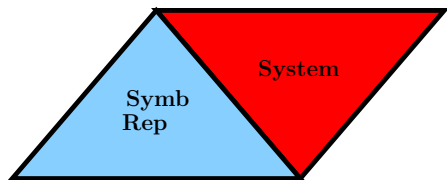
- Encode infinite sets of memory graphs by Presburger sets
- bisimulation between the pointer system and a counter system
- verification by FAST

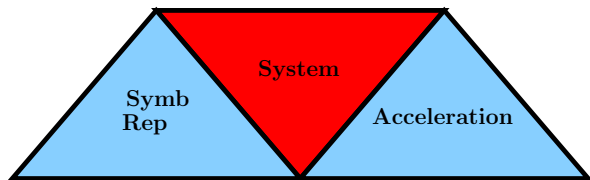
A prototype is in progress (with A. Sangnier and É. Lozes)

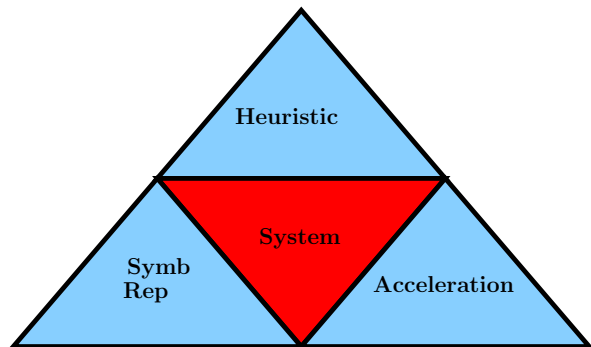
- Works well for ≈ 10 small standard examples
- Both qualitative and quantitative properties
- Allows to check programs with counters + pointers

- ➊ Introduction
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- ➍ Circuit selection
- ➎ the tool FAST
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1. Generic methodology [ATVA'05]

- unified acceleration framework
- power and limits (flattable systems)
- maximal circuit selection
- generic optimizations (reductions)

2. Instantiation to counter systems

- two acceleration algorithms
 - *[Finkel-Leroux 2002]*
 - [TACAS'04]
- *a reduction fit to counters [Finkel-Leroux 2002]*
- The tool FAST

3. Many experimentations

- Counter systems

- 40 infinite systems [CAV'03]
- TTP [TACAS'04]

- Counters + queues

- CES (in my PhD thesis, work with Laure Petrucci)
- *Stop and Wait Protocol* [Billington-Gallasch-Petrucci 2005]

- Pointer systems

- translation into counter systems [AVIS'06, AVIS'04]
- prototype, works on 10 standard examples (work with Étienne Lozes and Arnaud Sangnier)

Theoretical limitations (finite monoid, Presburger logic)

- not the main issue for protocols
- would be different for programs

Practical limitations:

1 Number of variables

- main point = complexity of relationship among variables
 - Example: Hand set with 50 variables is OK
 - Example: TTP2 with 20 variables is not
- no more than 100 variables

2 Number of transitions

- difficulty to find large circuits
- currently: circuits of length 4 with 20 variables

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A new version FAST_{ER} is released [CAV'06]

- A new architecture
 - Reachability set computation engine
 - Generic Presburger Interface
 - Presburger packages (LASH, MONA, OMEGA)
- A new Presburger package
 - Cache computation [Couvreur 2004]
 - (not yet optimized)
- New features in analysis
 - Circuit selection,
 - Convex acceleration

- Improve circuit detection: partial orders, system transformation
- Scale-up our methods: abstract-refine and checks methods
- Timed counter systems? (TPN, TA + counters, ...)